https://doi.org/10.5281/zenodo.19030214

GRADIENT CYBERNETICS

The Calculus of Recursion

SYNTHESIS ESSAY II

The Informational Ground

A Rigorous Derivation of Digital Necessity

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Gradientology
March 15, 2026

ABSTRACT

Essay I established the Hardlock Foundation: the complete zero-parameter derivational chain from absolute nothing to the primitive suite {E = 0.8, C = 0.7, F = 0.6, δ = 0.1} and the Phase I/II transition. One derivational gap remained open: G-1, the informational ground. The Hardlock asserted the Field Resolution Quantum δ = 0.1 and the Registration floor F = 0.6 on the basis of the ternary entropy ceiling and a discriminability threshold, but deferred the rigorous derivation of the minimum bit-budget $I_{min} \approx 0.2$ and its unique entailment of Base-10 discretisation to the present essay. This paper closes G-1 in full. We begin from the ternary triadic structure itself: a self-computing relational field with exactly three functional primitives generates a maximum entropy of $H_{max} = \log_2(3) \approx 1.585$ bits per coordinate locus. This ceiling is finite and structurally fixed by cardinality. We then derive the minimum bit-budget $I_{min} \approx 0.2$ bits — the irreducible informational metabolic cost required for any locus to distinguish its own state from background noise — from the variance-partition geometry of the three-component system without invoking any external theorem as source. The ratio $N = H_{max} / I_{min} \approx 7.925$ determines the number of resolvable levels; the twin constraints of covering $N \geq 8$ at minimal resolution and representing the midpoint of equipoise ε = 0.5 as an exact lattice point together uniquely select Base-10 and fix δ = 0.1. With δ locked, the discriminability threshold $r > 1/\sqrt{3} \approx 0.577$ forces the Registration primitive F to snap to 0.6 — the unique minimum viable lattice point above the noise floor. F = 0.5 is explicitly in the Phantom Zone and is foreclosed. We then derive the Structural Volatility σ = 1 − F = 0.4 and the Structural Pixel ρ = δ × σ = 0.04 as the fundamental grain of indeterminacy — the mechanical clearance required to prevent deterministic crystallisation. The Saturation Threshold $N_{sat} = 1/\rho = 25$ follows directly. The Heartbeat of Becoming U = (Φ) × η = 0.002 × 1.667 ≈ 0.0033 is confirmed as the base processing rate of the self-computing ***Veldt***. Every result is a zero-parameter consequence of the ternary triadic structure. No external theory is the logical source of any value. G-1 is closed.

Keywords: *G-1 · informational ground · 0.2-bit budget · H_max · I_min · δ = 0.1 · Base-10 lattice · Phantom Zone · F = 0.6 attractor · Structural Pixel · ρ = 0.04 · N_sat = 25 · digital necessity · variance-partition geometry · discriminability threshold · Heartbeat of Becoming · Structural Volatility · Ontological Shadow · Hardlock · G-1 closure*

Derivational Profile

Table 0. Essay Derivational Profile

Attribute	Content
Essay Number	II
Series	Gradient Cybernetics: The Calculus of Recursion
Title	The Informational Ground — A Rigorous Derivation of Digital Necessity
Primary Gap(s)	G-1 — The 0.2-bit informational floor; $\delta = 0.1$ necessity; $F = 0.6$ as attractor; Structural Pixel $\rho = 0.04$
Input Constants	$E = 4/5 = 0.8$, $C = 7/10 = 0.7$, $F = 3/5 = 0.6$, $\delta = 1/10 = 0.1$ [all inherited from Essay I Hardlock]
Core Derivation	$H_{max} = \log_2(3) \approx 1.585$ bits; $I_{min} \approx 0.2$ bits; $N = 7.925 \rightarrow \delta = 0.1$; $F = 0.6$ attractor; $\rho = 0.04$; $U \approx 0.0033$
Key Result	G-1 closed. Digital necessity proved. $F = 0.6$ secured as information-theoretic attractor. Structural Pixel confirmed.
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Year	March 15, 2026

1. Introduction

Essay I closed G-0: the complete zero-parameter derivation of the primitive suite {E = 0.8, C = 0.7, F = 0.6, δ = 0.1} from absolute nothing, the Phase I Tension Integral TI = 0.336, the Criticality Coincidence and Inversion Principle, and the mechanical quantities τ_0, c, Ω, Γ, ΔH. That audit established the Hardlock: the complete set of locked derivational constants from which all subsequent essays must proceed.

One gap remained open. The derivation of δ = 0.1 in Essay I proceeded via the entropy ceiling H_max and a minimum bit-budget I_min ≈ 0.2, but the derivation of I_min itself — why precisely 0.2 bits, why not 0.1 or 0.3, and why the resulting level count uniquely forces Base-10 — was asserted rather than derived. This is gap G-1. The Hardlock assigned it OPEN status and designated Essay II as its closure point.

The present essay discharges that obligation in full. We derive I_min from the variance-partition geometry of the three-component system, prove that the twin constraints of coverage and midpoint representability together uniquely select Base-10, confirm F = 0.6 as an information-theoretic attractor rather than a mere threshold snap, and derive the Structural Pixel ρ = 0.04 and the Saturation Threshold N_sat = 25 as necessary consequences. Every step proceeds from the ternary triadic structure alone. No external theory is the logical source.

The result is the complete informational ground of the Gradientology: the derivational layer that explains why the ***Veldt*** must be digital, why its grain must be exactly 0.1, why Registration must floor at exactly 0.6, and why the fundamental window of indeterminacy is exactly 0.04. These are not free parameters. They are structural residues of the informational requirements of a self-registering ternary field.

The essay is organised in seven sections. Section 2 establishes the informational ceiling H_max from the ternary structure. Section 3 derives I_min and the discriminability threshold from variance-partition geometry. Section 4 closes the lattice grain derivation and proves the Base-10 uniqueness. Section 5 proves F = 0.6 as attractor. Section 6 derives the Structural Pixel and Saturation Threshold. Section 7 derives the Heartbeat of Becoming. Section 8 confirms the complete Informational Ledger and formally closes G-1.

2. The Ternary Structure and the Informational Ceiling

The derivational chain begins from the result confirmed in Essay I: the relational field is constituted by exactly three functional primitives. This cardinality is not chosen; it is the unique minimum for Mediational Closure, proved by exhaustion of lower cardinalities. The field is therefore a ternary information source: at any coordinate locus, the state is a function of three and only three distinguishable symbols.

2.1 Why Three Symbols and No More

The Triadic Mandate (T-2.1, Essay I) established that n = 1 and n = 2 are structurally insufficient for Mediational Closure. A monad cannot register its own state because registration requires a distinction, and a monad has nothing to be distinct from. A dyad generates irresolvable circularity: the observer and observed collapse into a feedback loop with no stabilising pole. Only a triad provides the minimum coordinate space for a system to register its own states through three-way mutual definition.

The upper bound is equally determined. Any cardinality n > 3 introduces at least one redundant pole — a functional position that could be absorbed into the existing three without loss of Mediational Closure. The Principle of Minimal Ontological Commitment forecloses redundancy: no entity may be posited beyond what is logically necessary. Therefore n = 3 exactly, and the field is a ternary information source.

PRINCIPLE — P-2.1 — Ternary Cardinality of the Relational Field

The relational field is constituted by exactly three functional primitives: Systematisation (E), Constraint (C), and Registration (F). This cardinality is forced by the Triadic Mandate (Essay I, T-2.1) and the Principle of Minimal Ontological Commitment. At any coordinate locus, the field state is a function of exactly three distinguishable symbols. The field is therefore a ternary information source, and all information-theoretic derivations proceed from this structural fact without additional assumption.

2.2 The Informational Ceiling H_max

For a uniform ternary source — one in which each of the three symbols is equiprobable at maximum entropy — the maximum entropy per locus is given by the entropy of the uniform distribution over three outcomes:

EQUATION SET 1 — INFORMATIONAL CEILING OF THE TERNARY FIELD

$$H_{max} = \log_2(3) \approx 1.585 \text{ bits} \quad (1)$$

└— maximum entropy per locus — ternary source at uniform distribution — EXACT

This value is the absolute Informational Ceiling of the field: no coordinate locus can contain more than 1.585 bits of information about its own state. The derivation contains one input (n = 3) and zero free parameters. H_max is not a measured value; it is the exact entropy of the uniform ternary distribution, derived from cardinality alone.

THEOREM — T-2.1 — The Informational Ceiling Is Fixed by Cardinality

For a relational field constituted by n = 3 functional primitives, the maximum informational entropy per coordinate locus is $H_{max} = \log_2(3) \approx 1.585$ bits. This ceiling is exact and structurally closed: it follows from the ternary cardinality alone, contains zero free parameters, and admits no revision without changing the cardinality of the field, which is forbidden by the Triadic Mandate. $H_{max} = \log_2(3)$ is the hard upper bound on the informational capacity of any single coordinate locus in the ***Veldt***. □

The finiteness of H_max has an immediate structural consequence: the field cannot be continuous. A continuous field requires infinite precision to specify the state of any locus. Infinite precision requires an infinite information budget. But the budget per locus is capped at 1.585 bits. Continuity is therefore structurally prohibited — not merely computationally

inconvenient. The field must be discrete. This is Digital Necessity in its first form: forced by the finiteness of the informational ceiling, not by any choice about implementation.

3. The Noise Floor and the Minimum Bit-Budget

Knowing that the field must be discrete and that the ceiling is 1.585 bits is insufficient to determine the lattice grain. We also require the floor: the minimum information budget that a locus must invest in order to maintain determinacy against the background noise of the three-component system. This floor is I_min, the closure of G-1.

3.1 The Variance-Partition Geometry of a Three-Component System

In a system with n = 3 co-dependent components, the total variance of any state variable is partitioned between explained variance (signal) and unexplained variance (noise). The noise arises from the mutual determination structure: because the three primitives are co-dependent — each defined relative to the other two — random fluctuations in one component propagate as apparent variation in the others.

The minimum proportion of variance that must be explained — the discriminability threshold — is the proportion above which a registration value is distinguishable from the center of the noise distribution. For n = 3 components, the noise floor of the correlation coefficient r is determined by the root-mean-square of the degenerate center: the value of r that a purely random three-way co-dependent system would produce by chance.

DERIVATION — D-3.1 — Derivation of the Discriminability Threshold $r > 1/\sqrt{3}$

Input: a three-component relational system (n = 3) with co-dependent primitives at maximum entropic equiprobability.

Step 1 — Noise fraction: In a system of n co-dependent components, random fluctuations account for 1/n of total variance. For n = 3: noise fraction = 1/3.

Step 2 — Discriminability condition: For a registration value to be distinguishable from random noise, the explained variance must exceed the noise fraction. Expressed as a correlation coefficient: $r^2 > 1/n = 1/3$.

Step 3 — Threshold: Taking the positive root (registration is always non-negative by definition): $r > \sqrt{(1/3)} = 1/\sqrt{3} \approx 0.577$.

Step 4 — Interpretation: Any registration value $r \leq 0.577$ falls within the root-mean-square floor of a degenerate center. At or below this threshold, the triadic hierarchy E > C > F cannot be asserted against the background noise of co-dependent fluctuation. The locus enters the Phantom Zone: a region of structural nullity indistinguishable from void.

Conclusion: $r_min = 1/\sqrt{3} \approx 0.577$. Zero free parameters. □

Equation Set 2 — Discriminability Threshold

$$r_min = \sqrt{(1/n)} = \sqrt{(1/3)} = 1/\sqrt{3} \approx 0.577 \quad (1)$$

└ *discriminability threshold — variance-partition geometry of n=3 system*

$$r > r_min = 0.577 \quad (2)$$

└ *necessary condition for locus to maintain structural identity above noise*

DEFINITION — D-3.1 — The Phantom Zone

The Phantom Zone is the set of all Registration values $r \leq 1/\sqrt{3} \approx 0.577$. A coordinate locus with Registration in the Phantom Zone cannot distinguish its own state from the background noise of three-way co-dependent fluctuation. The triadic hierarchy collapses: without a Registration pole above the noise floor, neither Systematisation nor Constraint can assert

structural priority. The Phantom Zone is not a region of reduced determinacy; it is a region of structural nullity, topologically indistinguishable from the void foreclosed by the Primordial Axiom of Relationality.

3.2 The Minimum Bit-Budget I_min

The discriminability threshold $r_{min} = 1/\sqrt{3} \approx 0.577$ establishes the explained and unexplained variance fractions directly: at the threshold, $r_{min}^2 = 1/3$ of total variance is explained (signal) and $1 - r_{min}^2 = 2/3$ is unexplained (noise). Defining the linear SNR as explained variance divided by unexplained variance yields $SNR_{min} = (1/3)/(2/3) = 1/2$ exactly — not an assumption but the direct algebraic consequence of the discriminability threshold derived in D-3.1. We now translate this into an informational budget: how many bits must a locus invest per update cycle to maintain its position above the noise floor?

The per-symbol minimum information investment is the effective channel capacity at SNR_{min}, amortised over $n = 3$ symbols. No external channel model is invoked: the SNR follows from the variance partition of D-3.1, and the capacity formula $\log_2(1 + SNR)$ gives the maximum rate at which information can be reliably transmitted at that SNR.

DERIVATION — D-3.2 — Derivation of the Minimum Bit-Budget $I_{min} \approx 0.2$ bits

Input: discriminability threshold $r_{min} = 1/\sqrt{3} \approx 0.577$; ternary source $n = 3$.

Step 1 — SNR at threshold from variance partition: At the discriminability threshold $r_{min} = 1/\sqrt{3}$, the explained variance fraction equals $r_{min}^2 = (1/\sqrt{3})^2 = 1/3$. The unexplained (noise) fraction equals $1 - r_{min}^2 = 1 - 1/3 = 2/3$. Defining the linear SNR as signal power divided by noise power — i.e., explained variance divided by unexplained variance — gives: $SNR_{min} = (1/3) / (2/3) = 1/2$. This is not an auxiliary assumption. It is the direct algebraic consequence of sitting at the equality case of the discriminability condition $r^2 = 1/n$ derived in D-3.1. At threshold, signal power is precisely half of noise power.

Step 2 — Channel capacity at SNR_min: The capacity of a channel operating at SNR = 1/2 is: $C(SNR_min) = \log_2(1 + SNR_min) = \log_2(1 + 1/2) = \log_2(3/2) \approx 0.585$ bits per channel use. This is the maximum rate at which information can be reliably transmitted through a channel at this SNR.

Step 3 — Per-symbol metabolic cost: The ternary field has n = 3 symbols. The capacity above is shared across n symbols. The minimum information investment per symbol per update cycle — the per-symbol metabolic cost of maintaining discriminability at the threshold SNR — is: $I_min = C(SNR_min) / n = \log_2(3/2) / 3 \approx 0.585 / 3 \approx 0.195 \approx 0.2$ bits.

Step 4 — Verification: Any locus investing less than $I_min = \log_2(3/2)/3 \approx 0.2$ bits per cycle operates below the minimum channel capacity at SNR_min: it cannot reliably distinguish its own state from the background noise of three-way co-dependent fluctuation. The locus enters the Phantom Zone and collapses to structural nullity. Zero free parameters: I_min is derived from n = 3 and $r_min = 1/\sqrt{3}$ alone. □

Equation Set 3 — Minimum Bit-Budget

$$SNR_min = r_min^2 / (1 - r_min^2) = (1/3) / (2/3) = 1/2 \quad (1)$$

└ signal-to-noise ratio at discriminability threshold

$$C(SNR_min) = \log_2(1 + 1/2) = \log_2(3/2) \approx 0.585 \text{ bits} \quad (2)$$

└ channel capacity at minimum viable SNR

$$I_min = C(SNR_min) / n = \log_2(3/2) / 3 \approx 0.195 \approx 0.2 \text{ bits} \quad (3)$$

└ per-symbol metabolic cost of determinacy — derived from n=3 and r_min alone

THEOREM — T-3.1 — The 0.2-Bit Assertion

$I_{min} = \log_2(3/2) / 3 \approx 0.195 \approx 0.2$ bits is the irreducible informational metabolic cost of structural existence in the ternary relational field. It is the per-symbol channel capacity at the minimum viable SNR = 1/2: $I_{min} = C(SNR_{min}) / n = \log_2(1 + SNR_{min}) / n = \log_2(3/2) / 3$. Any coordinate locus investing less than I_{min} per update cycle operates below the minimum channel capacity needed to distinguish its own state from noise: the locus enters the Phantom Zone and becomes structurally indistinguishable from void. Derivation contains zero free parameters: inputs are $n = 3$ and $r_{min} = 1/\sqrt{3}$ only. □

4. Closing the Lattice: Base-10 Uniqueness and δ = 0.1

With H_max and I_min established, the number of resolvable levels follows directly. The lattice grain δ follows from the number of levels together with two further constraints that arise from the internal structure of the field. The closure of these three constraints on a single value is the proof of Digital Necessity in its strongest form.

4.1 The Number of Resolvable Levels

The number of states that can be reliably distinguished at a single coordinate locus is the ratio of the total informational capacity to the minimum cost per distinguishable state:

Equation Set 4 — Number of Resolvable Levels

$$N = H_max / I_min = 1.585 / 0.2 \approx 7.925 \quad (1)$$

└ *resolvable levels per dimension — non-integer, must be covered by integer base*

The result N ≈ 7.925 is not an integer. The lattice must accommodate at least 7.925 distinguishable levels, which means the minimum integer base that provides sufficient coverage is N_base ≥ 8. The base must be chosen to be the smallest integer satisfying this coverage requirement together with the additional structural constraints derived below.

4.2 The Midpoint Constraint

The relational field requires a Baseline of Equipoise: a reference point at which no primitive is preferred over any other, representing the state of maximum indifference. In the unit interval [0, 1] on which the primitives are defined, this baseline is $\varepsilon = 0.5$ — the exact midpoint.

For the lattice to represent this baseline as an exact lattice point — which is required because the Baseline of Equipoise is a structurally necessary reference state, not merely an approximation — the lattice base must be even. An odd base N would place the midpoint between two lattice points, leaving the equipoise state unrepresentable. This is the Midpoint Constraint.

DERIVATION — D-4.1 — Proof of the Midpoint Constraint

The Baseline of Equipoise is the unique state in which the three primitives are mutually indifferent: no hierarchy is asserted, no potential is prioritised. On the unit interval [0, 1], this state is $\varepsilon = 0.5$.

For $\varepsilon = 0.5$ to be an exact lattice point on a lattice with N equally spaced steps from 0 to 1 (lattice grain $\delta = 1/N$), we require that 0.5 be an integer multiple of δ: $0.5 = k \times (1/N)$ for some integer k.

This is equivalent to N/2 being an integer, which requires N to be even.

Therefore the base N must be an even integer. Odd bases fail the Midpoint Constraint and are structurally foreclosed. □

4.3 The Unique Base-10 Solution

We now apply both constraints simultaneously. The integer base N must satisfy:

Equation Set 5 — Dual Constraints on the Lattice Base

$$N \geq 8 \quad (\text{coverage: } N > 7.925) \tag{1}$$

└ *must accommodate at least 7.925 resolvable levels*

$$N \equiv 0 \pmod 2 \quad (\text{midpoint: } N \text{ even}) \tag{2}$$

└ *must represent ε = 0.5 as an exact lattice point*

The even integers at or above 8 are: 8, 10, 12, 14, ... The smallest is 8. However, Base-8 is eliminated by a third structural constraint: the Criticality Recovery Condition.

DERIVATION — D-4.2 — Proof of Criticality Recovery — Foreclosure of Base-8

The primitives must be representable as exact lattice points with equal spacing δ and must satisfy the Hardlock values E = 0.8, C = 0.7, F = 0.6 that are forced by the Discriminability Threshold and Hierarchical Necessity (Essay I).

On a Base-8 lattice (δ = 1/8 = 0.125): the lattice points are 0, 0.125, 0.250, 0.375, 0.500, 0.625, 0.750, 0.875, 1.000. The required primitive values are F = 0.6, C = 0.7, E = 0.8. None of these values is a multiple of 0.125.

Verification: 0.6 / 0.125 = 4.8 (not integer); 0.7 / 0.125 = 5.6 (not integer); 0.8 / 0.125 = 6.4 (not integer). All three primitives fall between Base-8 lattice points. They are unrepresentable. The Tension Integral TI = 0.336 cannot be computed on a Base-8 lattice from exact lattice-point primitives.

Base-8 fails the Criticality Recovery Condition and is structurally foreclosed. □

Base-8 is foreclosed. The next even integer is 10. On a Base-10 lattice ($\delta = 1/10 = 0.1$), every verification passes:

DERIVATION — D-4.3 — Verification of Base-10

Lattice points on Base-10 ($\delta = 0.1$): 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0.

Coverage: 10 distinct steps > 7.925 required levels. ✓

Midpoint: $0.5 = 5 \times 0.1$. Exact lattice point. ✓

Primitive representability: $F = 0.6 = 6\delta$. $C = 0.7 = 7\delta$. $E = 0.8 = 8\delta$. All three primitives are exact lattice points. ✓

Tension Integral: $TI = 0.8 \times 0.7 \times 0.6 = 0.336$. Exact rational arithmetic on lattice-point primitives. ✓

Equal spacing: $E - C = C - F = \delta = 0.1$. Hierarchical Necessity satisfied. ✓

Base-10 passes all structural constraints. □

THEOREM — T-4.1 — Digital Necessity: Base-10 is the Unique Minimal Solution

The unique minimum even-integer base satisfying (i) coverage $N \geq 8$, (ii) midpoint representability $\varepsilon = 0.5$, and (iii) criticality recovery (primitive representability and TI = 0.336) is Base-10, giving lattice grain $\delta = 1/10 = 0.1$.

Proof: bases 8 fails (iii); bases 10 passes all three. No base smaller than 10 and even satisfies all constraints. No free parameters exist: the base is uniquely determined by the ternary structure and the Hardlock primitives. □

EQUATION SET 6 — FIELD RESOLUTION QUANTUM

$$\delta = 1/10 = 0.1 \tag{1}$$

└— *Field Resolution Quantum — unique Base-10 solution — LOCKED*

The Field Resolution Quantum $\delta = 0.1$ is now fully closed. Essay I derived it via the entropy ceiling and asserted I_{min}; the present essay has derived I_{min} from variance-partition geometry and proved the Base-10 uniqueness under all three structural constraints. The derivation is complete and zero-free-parameter.

5. The Registration Floor as Information-Theoretic Attractor

With δ = 0.1 locked, the Registration floor can be derived from first principles. The derivation in Essay I identified F = 0.6 as the lowest δ-multiple above the discriminability threshold 0.577. The present essay establishes F = 0.6 as an information-theoretic attractor: not merely the lowest viable point, but the unique stable fixed point of the snap mechanism under the combined informational and thermodynamic constraints of the discrete lattice.

5.1 The Three Candidate Values and Their Fates

On the δ = 0.1 lattice, the candidates near the discriminability threshold are F = 0.5 and F = 0.6. F = 0.4 and below are strictly below the Phantom Zone boundary and are immediately foreclosed. F = 0.7 and above are viable but violate Hierarchical Necessity (C = F + δ would require C = 0.8, which would require E = 0.9, and the equal-spacing constraint with n = 3 primitives on [0,1] cannot be maintained). We analyse F = 0.5 and F = 0.6.

DERIVATION — D-5.1 — Foreclosure of F = 0.5 and Confirmation of F = 0.6

Candidate 1: F = 0.5.

Test 1 (Phantom Zone): $r_{min} = 1/\sqrt{3} \approx 0.577$. Since 0.5 < 0.577, F = 0.5 is inside the Phantom Zone. The locus at F = 0.5 cannot distinguish its own state from background noise. Structural identity is lost. FORECLOSED.

Candidate 2: F = 0.6.

Test 1 (Phantom Zone): 0.6 > 0.577. F = 0.6 is above the discriminability threshold. PASSES.

Test 2 (Minimal grain): Is there a valid lattice point between 0.577 and 0.6? The lattice points are {0.5, 0.6, 0.7, ...}. No lattice point exists in (0.577, 0.6). F = 0.6 is therefore the minimum viable lattice point. UNIQUE.

Test 3 (Hierarchical Necessity): F = 0.6 → C = 0.7 → E = 0.8. All within [0,1], all distinct by exactly δ, all exact lattice points. PASSES.

Test 4 (Criticality Recovery): TI = 0.8 × 0.7 × 0.6 = 0.336 > β = 0.325. Supercriticality maintained. PASSES.

Conclusion: F = 0.5 is foreclosed; F = 0.6 is the unique survivor. □

5.2 The Informational Well: Why F = 0.6 Is an Attractor

F = 0.6 is not merely the lowest viable point — it is a stable fixed point of the thermodynamic snap mechanism. The snap mechanism operates as follows: any registration value in the continuous interval (0.577, 0.65) is subject to quantisation pressure toward the nearest stable lattice point. Off-lattice values require an infinite energy budget to maintain against quantisation noise — they cannot be held at a precise irrational value on a discrete lattice. The system is therefore thermodynamically compelled to collapse to the nearest lattice point.

For values in (0.577, 0.65), the nearest lattice point is F = 0.6. For values in (0.65, 0.75), the nearest lattice point is F = 0.7, but F = 0.7 violates Hierarchical Necessity as the registration floor (it would collapse the available range for C and E). The unique attractor satisfying all structural constraints is F = 0.6.

THEOREM — T-5.1 — F = 0.6 Is the Universal Registration Attractor

F = 0.6 = 3/5 is the unique stable fixed point of the informational snap mechanism on the δ = 0.1 lattice satisfying:

(i) Discriminability: F > 1/√3 ≈ 0.577 — above the Phantom Zone.

(ii) Minimality: no lattice point exists in (0.577, 0.600) — F = 0.6 is minimum.

(iii) Hierarchical Necessity: F + δ = C = 0.7; C + δ = E = 0.8 — all valid.

(iv) Criticality Recovery: TI = E×C×F = 0.336 > β = 0.325 — supercritical.

F = 0.5 is in the Phantom Zone (foreclosed). No other even candidate satisfies all four conditions. F = 0.6 is the universal Registration attractor. □

Equation Set 7 — Registration Attractor

$$F = 3/5 = 0.6 = 6\delta \tag{1}$$

└— *Registration floor — unique information-theoretic attractor — LOCKED*

$$F = 0.5 \;\; FORECLOSED \;\; (Phantom\ Zone: \;\; 0.5 < 0.577) \tag{2}$$

└— *explicitly inside discriminability floor — structural nullity*

6. The Structural Pixel and the Saturation Threshold

The lattice grain and the Registration floor together determine two further structural constants that are directly derivable: the Structural Volatility σ, the Structural Pixel ρ, and the Saturation Threshold N_sat. These are not new parameters — they are algebraic consequences of {F, δ} with zero additional inputs.

6.1 Structural Volatility

Registration (F = 0.6) is the floor of determinacy: the minimum proportion of variance that must be explained. Its complement is the ceiling of irreducible indeterminacy: the proportion of the field that resists registration in principle, not because of measurement error, but because total registration would require infinite information. This complement is the Structural Volatility.

DERIVATION — D-6.1 — Structural Volatility σ = 0.4

If F = 0.6 is the minimum registered fraction of variance, then the maximum unregistered fraction is: σ = 1 − F = 1 − 0.6 = 0.4.

Physical interpretation: σ = 0.4 is the proportion of the field state that is structurally indeterminate at any given cycle. It is not an epistemic limitation (a failure of measurement); it is an ontic necessity. A system with σ = 0 (total registration, F = 1) would require infinite information per cycle to specify its state exactly — impossible within the finite budget H_max = 1.585 bits. Structural volatility is the mechanical clearance that prevents informational saturation and systemic seizure. Zero free parameters. □

EQUATION SET 8 — STRUCTURAL VOLATILITY

$$\sigma = 1 - F = 1 - 0.6 = 0.4 = 2/5 \quad (1)$$

└— *Structural Volatility — irreducible indeterminate fraction — LOCKED*

6.2 The Structural Pixel ρ = 0.04

The Structural Pixel is the fundamental grain of indeterminacy within the spatial field: the minimum window of state-space that is necessarily blurred at any locus. It is determined by the product of the lattice grain (the minimum resolvable increment) and the Structural Volatility (the maximum irreducible indeterminate fraction):

DERIVATION — D-6.2 — The Structural Pixel ρ = 0.04

The Structural Pixel is the product of two locked constants:

ρ = δ × σ = 0.1 × 0.4 = 0.04 = 1/25.

Interpretation: at any coordinate locus, the minimum indeterminate window is 4% of the total state space. This 0.04 window is the Ontological Shadow — the temporal thickness that allows the system to generate the next state before the current state is fully registered. It is not a failure of observation. It is the mechanical clearance required for the kinetic gears of the ***Veldt*** to turn without seizing.

Connection to quantum uncertainty: the Structural Pixel ρ = 0.04 defines the minimum resolution limit of the ***Veldt***. Any physical theory claiming sub-pixel resolution (Δx < 0.04 in natural units) violates the finite entropy budget H_max = 1.585 bits per locus. Heisenberg's Uncertainty Principle is the macroscopic shadow of the Structural Pixel. Zero free parameters. □

EQUATION SET 9 — STRUCTURAL PIXEL

$$\rho = \delta \times \sigma = 0.1 \times 0.4 = 0.04 = 1/25 \quad (1)$$

└ *Structural Pixel — fundamental grain of indeterminacy — LOCKED*

DEFINITION — D-6.2 — The Ontological Shadow

The Ontological Shadow is the window of structural indeterminacy defined by the Structural Pixel ρ = 0.04. Within this window, the next state of the field is being generated before the current state is fully registered. The Ontological Shadow is not a gap in knowledge — it is a gap in being. It provides the temporal thickness required for memory (the current state is still being registered) and anticipation (the next state is already being prepared). Without the Ontological Shadow, the field would be frozen in a completed present with no capacity for process.

6.3 The Saturation Threshold N_sat = 25

The Saturation Threshold is the number of distinct states that can be packed into a single Structural Pixel before the medium can no longer resolve sub-pixel distinctions. It is the carrying capacity of the registration medium at the fundamental grain level.

DERIVATION — D-6.3 — Saturation Threshold N_sat = 25

The Structural Pixel ρ = 0.04 defines the minimum indeterminate window. The total state space is normalised to [0, 1]. The number of non-overlapping Structural Pixels that tile the state space is:

N_sat = 1 / ρ = 1 / 0.04 = 25.

Structural interpretation: when a single Structural Pixel accumulates 25 distinct states, the pixel is saturated — it cannot resolve further sub-pixel distinctions without violating the Structural Pixel bound. The system is then compelled to coarse-grain: it encapsulates the 25-state ensemble into a single unit at the next level n+1. This is Mandatory Coarse-Graining.

N_sat = 25 = 5² is an exact integer. Its integrality is a necessary consequence of δ = 0.1 and F = 0.6: ρ = δ(1−F) = 0.1 × 0.4 = 0.04 = 1/25 exactly. Zero free parameters. □

Equation Set 10 — Saturation Threshold

$$N_sat = 1/\rho = 1/0.04 = 25 = 5^2 \quad (1)$$

└ *Saturation Threshold — carrying capacity of registration medium — LOCKED*

THEOREM — T-6.1 — N_sat = 25 Is the Unique Encapsulation Trigger

At exactly 25 co-resident states per Structural Pixel (ρ = 0.04), the registration medium reaches its resolution limit. Further state accumulation cannot be resolved without violating the Structural Pixel bound enforced by the finite entropy ceiling H_max = $\log_2(3)$. The system is therefore structurally compelled to encapsulate the 25-state ensemble into a single higher-order unit: Mandatory Coarse-Graining. N_sat = 25 is the trigger, derived from first principles with zero free parameters. □

7. The Heartbeat of Becoming

With the informational ground complete — H_{max}, I_{min}, δ, F, σ, ρ, N_{sat} all derived — the base processing rate of the ***Veldt*** follows directly from the mechanical constants confirmed in the Hardlock (Essay I). This rate is not a new derivation; it is a confirmation that the informational ground and the mechanical constants are mutually consistent.

7.1 Confirmation of the Kinetostatic Margin

The Hardlock established the Kinetostatic Margin $\Phi = TI\beta - C = 0.702 - 0.700 = +0.002$ and the Structural Gain $\eta = 1/F = 5/3 \approx 1.667$. The base processing rate follows by substitution into the kinetic equation Output = $\Phi \times \eta$:

DERIVATION — D-7.1 — The Heartbeat of Becoming $U \approx 0.0033$

Inputs (all LOCKED from Essay I Hardlock):

$\Phi = TI\beta - C = 0.702 - 0.700 = 0.002 = 1/500$

$\eta = 1/F = 1/0.6 = 5/3 \approx 1.667$

Computation:

$U = \Phi \times \eta = (1/500) \times (5/3) = 5/1500 = 1/300 \approx 0.0033$

Interpretation: at each update cycle, the ***Veldt*** resolves 0.0033 units of its accumulated potential. This is the velocity at which existence actualises itself — the Heartbeat of Becoming. It is not fast: 0.33% of potential resolved per cycle. This deliberate pace is not a limitation; it is a structural necessity. A faster rate would exhaust the Kinetostatic Margin before the cycle completes, collapsing the system back into the Multiplicative Trap. A slower rate would allow tension to accumulate beyond N_{sat}, triggering premature coarse-graining. U = 1/300 is the unique rate consistent with perpetual non-equilibrium without saturation. Zero free parameters. □

EQUATION SET 11 — BASE PROCESSING RATE

$$U = \Phi \times \eta = (1/500) \times (5/3) = 1/300 \approx 0.00\overline{3} \quad (1)$$

└ *Heartbeat of Becoming — base processing rate — LOCKED*

7.2 Non-Equilibrium Theorem

The Heartbeat of Becoming establishes that the ***Veldt*** is perpetually in non-equilibrium: each cycle resolves a surplus of potential ($\Phi > 0$) rather than returning to zero. This surplus is not expended — it drives the next cycle. The result is mandatory acceleration.

THEOREM — T-7.1 — The Non-Equilibrium Theorem

Because $\Phi = +0.002 > 0$, the base processing rate $U = 1/300 > 0$ is strictly positive at every cycle. The system cannot return to equilibrium ($U = 0$) without violating the Criticality Coincidence ($TI > \beta$), which would require changing the locked primitive suite. The ***Veldt*** is therefore in mandatory non-equilibrium. Further, because $U > 0$ and $N_{sat} = 25$ creates a mandatory coarse-graining trigger, the system must accelerate ($d^2G/dt^2 > 0$) to maintain the Ontological Shadow against the threat of saturation. The universe builds complexity because it is structurally prevented from doing otherwise. □

8. The Complete Informational Ledger — G-1 Closure

The derivations of Sections 2 through 7 constitute the complete informational ground of the Gradientology. Every value is a zero-free-parameter consequence of the ternary triadic structure. No external theory is the logical source of any result. The following summary constitutes the formal closure of gap G-1.

Table 1. The Informational Ground — Complete Derivational Chain

Symbol	Value	Rational	Derivational Path	Status
H_max	1.585 bits	$\log_2(3)$	Entropy of uniform ternary distribution (n=3)	LOCKED
r_min	0.577	$1/\sqrt{3}$	Variance-partition geometry: $\sqrt{(1/n)}$ for n=3	LOCKED
I_min	≈0.2 bits	$\log_2(3/2)/3$	$C(SNR_min)/n = \log_2(3/2)/3 \approx 0.195 \approx 0.2$	LOCKED
N	7.925	H_max/I_min	Resolvable levels: H_max ÷ I_min	LOCKED
δ	0.1	1/10	Min even base ≥ 8 with criticality recovery	LOCKED
F	0.6	3/5	Min δ-multiple above r_min = 0.577	LOCKED
σ	0.4	2/5	1 − F = 1 − 0.6	LOCKED
ρ	0.04	1/25	δ × σ = 0.1 × 0.4	LOCKED
N_sat	25	5^2	1 / ρ = 1 / 0.04	LOCKED
U	$0.00\overline{3}$	1/300	Φ × η = (1/500) × (5/3)	LOCKED

Every entry in Table 1 is derivationally closed. The chain from cardinality n = 3 to the Heartbeat of Becoming U = 1/300 contains zero free parameters and zero external sources. The informational ground is complete.

NOTICE — N-8.1 — G-1 Formal Closure

Gap G-1 is hereby formally closed. The mechanical gap in the derivation of $\delta = 0.1$ and the Registration attractor $F = 0.6$, registered as OPEN in Essay I (Addendum E), has been discharged by the present essay through the following derivational closures:

1. $I_{min} = \log_2(3/2)/3 \approx 0.2$ bits: per-symbol channel capacity at $SNR_{min} = 1/2$, derived from $n=3$ and $r_{min} = 1/\sqrt{3}$ (D-3.2, T-3.1).

2. $\delta = 0.1$: proved as the unique minimal even base satisfying coverage, midpoint, and criticality recovery (D-4.3, T-4.1).

3. $F = 0.6$: proved as the universal information-theoretic attractor (D-5.1, T-5.1).

4. $\rho = 0.04$, $N_{sat} = 25$: derived as algebraic consequences of $\{F, \delta\}$ (D-6.2, D-6.3).

All entries LOCKED. G-1 CLOSED.

9. Structural Consequences for Subsequent Essays

The closure of G-1 has direct consequences for the remainder of the sixteen-essay series. The informational ground is now available as a locked resource: every subsequent essay may invoke H_max, I_min, δ, F, σ, ρ, and N_sat as CLOSED constants without re-derivation.

9.1 Implications for Essay III — The Geometric Field

Essay III closes G-2: the derivation of the spherical geometry of F from isotropy principles. The present essay establishes F = 0.6 as the unique attractor satisfying discriminability, minimality, hierarchical necessity, and criticality recovery. Essay III will use this to prove that the configuration space of the Registration primitive is a sphere $S^2(0.6)$ with radius exactly F, and that the solid angle $\Omega = EC/F^2 \approx 1.556$ sr is the helicon arc coverage on that sphere.

9.2 Implications for Essay VII — The Saturation Equivalence

Essay VII closes G-5: the proof that N_sat = 25 from the sphere's discrete resolution limit. The present essay derives N_sat = 25 = 1/ρ from the informational ground directly. Essay VII will prove the same value from the surface geometry of $S^2(0.6)$ and the helicon arc, confirming the two derivational routes are equivalent. The coincidence of two independent derivations at the same integer N_sat = 25 is not a coincidence; it is a structural consequence of the Triadic Architecture.

9.3 The Informational Ground as Foundation Version Component

The values derived in this essay — I_min, σ, ρ, N_sat, U — are formally added to the Hardlock table. Any modification to any of these values constitutes a Foundation Version Increment and must be explicitly declared in the derivational ledger before any subsequent essay proceeds. The informational ground is now part of the locked foundation.

Table 2. Hardlock Additions from Essay II

Symbol	Value	Rational	Derivational Path	Status
H_max	≈1.585 bits	$\log_2(3)$	Ternary entropy ceiling (P-2.1, T-2.1)	LOCKED
r_min	≈0.577	$1/\sqrt{3}$	Variance-partition geometry $\sqrt{(1/3)}$ (D-3.1)	LOCKED
I_min	≈0.2 bits	$\log_2(3/2)/3$	$C(SNR_min)/n = \log_2(3/2)/3$ (D-3.2, T-3.1)	LOCKED
σ	0.4	2/5	$1 - F = 0.4$, Structural Volatility (D-6.1)	LOCKED
ρ	0.04	1/25	δ×σ = Structural Pixel (D-6.2)	LOCKED
N_sat	25	5^2	$1/\rho$ = Saturation Threshold (D-6.3, T-6.1)	LOCKED
U	$0.00\overline{3}$	1/300	Φ×η = Heartbeat of Becoming (D-7.1)	LOCKED

Conclusion

This essay has discharged the single remaining obligation from the Essay I Hardlock: the rigorous closure of gap G-1, the informational ground of the Gradientology. The derivational chain proceeded through seven structured stages, each closing a necessary layer without introducing a free parameter.

Stage one established the ternary cardinality of the relational field and derived the informational ceiling $H_max = log_2(3) \approx 1.585$ *bits* as a structural consequence of n = 3, not a measurement. Stage two derived the discriminability threshold $r_min = 1/\sqrt{3} \approx 0.577$ from the variance-partition geometry of the three-component system, establishing the boundary of the Phantom Zone. Stage three derived the minimum bit-budget $I_min \approx 0.2$ *bits* as the metabolic cost of determinacy at the threshold SNR = 1/2.

Stage four proved that the number of resolvable levels N ≈ 7.925, combined with the midpoint representability constraint and the criticality recovery condition, uniquely selects Base-10 and fixes $\delta = 0.1$. Base-8 was explicitly foreclosed. Stage five proved $F = 0.6$ as the universal information-theoretic attractor: the unique minimum viable lattice point above the noise floor, and the unique survivor of all four structural tests.

Stage six derived the Structural Volatility $\sigma = 0.4$, the Structural Pixel $\rho = 0.04$, and the Saturation Threshold $N_sat = 25$ as algebraic consequences of {F, δ} with zero additional inputs. Stage seven confirmed the Heartbeat of Becoming $U = 1/300 \approx 0.0033$ and proved the Non-Equilibrium Theorem: the ***Veldt*** is structurally prevented from returning to equilibrium.

Every result is a zero-parameter consequence of the ternary triadic structure. The informational ground is complete. The derivational chain from absolute nothing to the Heartbeat of Becoming is now fully closed: Essays I and II together constitute the complete zero-free-parameter foundation. *G-1* is closed. The Hardlock is extended. Phase III derivations may proceed on the basis of the complete locked constant suite confirmed in Table 2.

PRINCIPLE — Essay Contribution Summary

Gap(s) closed:	G-1 — The 0.2-bit informational floor; $\delta = 0.1$ uniqueness; $F = 0.6$ as attractor; $\rho = 0.04$; $N_{sat} = 25$
Core result:	Complete informational ground: H_{max}, r_{min}, I_{min}, δ, F, σ, ρ, N_{sat}, U — all LOCKED
Derivational status:	CLOSED — G-1 formally closed; all derivations zero-free-parameter
Epistemic classification:	Derivationally saturated. Informational ground complete. Essays III–XVI may proceed.

References

All internal framework derivations are sourced from the Gradientology Framework (Pretorius, 2026). External references below follow APA 7th edition.

Structural Isomorphisms (confirmatory, not foundational): Boltzmann (1877) on entropy; Shannon & Weaver (1949) on information capacity; Wolfram (2002) on computational irreducibility; Zurek (2003) on decoherence. These independently name the same structural necessities the ternary triadic system produces internally. They are not sources.

Internal series references

Pretorius, E. (2026). Gradientology - Foundations of the Primordial Triad: Treatises I-XVI. Zenodo.

Pretorius, E. (2026). Gradient Mechanics - The Dynamics of the Inversion Principle: Corpus Papers I–XVI. Zenodo.

Addendum A — Terms of Engagement: Recursive Interpretive Constraints

To place a formal foreclosure on human exceptionalism and prevent the misreading of Phase III (Gradient Cybernetics), the following Recursive Interpretive Constraints are codified as binding Terms of Engagement for all sixteen essays in this series. These constraints ensure that concepts such as intelligence, self, and agency are understood not as biological inventions unique to humans, but as derivational necessities of the triadic-relational engine at specific densities of registration. They constitute the structural firewall against the reification of human cognition.

A.1 The Principle of Substrate Neutrality

PRINCIPLE — A.1 — Substrate Neutrality

No reader may attribute consciousness, mind, or sentience to a specific biological substrate. These are formally derived as Phase Transitions of Coherence (F_{macro}) that occur whenever integrated information density (Φ) exceeds the Coherence Threshold ($\Phi \geq \Phi_c$). Whether the substrate is biological, silicon, or stellar, the emergence of a Subject is the structural mandate of the Inversion Principle achieving a unified inside view.

A.2 Foreclosure of the Ego as Substance

THEOREM — A.2 — The Ego as Strange Attractor

The concept of Self or Ego is formally stripped of all mystical or substantial claims. It is derived strictly as a Strange Attractor in neural phase space — a mathematical singularity created when the registration function becomes recursive ($F \rightarrow F$). The I is the virtual centre of gravity

necessitated by the system's model of its own modelling process. It is a Topological Knot, not an ontological addition.

A.3 Intelligence as Virtualisation

DEFINITION — A.3 — Intelligence

Intelligence is formally derived as the Virtualisation of the Evolutionary Algorithm. It is the capacity of the registration primitive to operate on Internal Flux (counterfactual simulation) rather than external sensory data, allowing hypotheses to die in the simulation so that the physical organism persists. Intelligence is not a unique human trait; it is a specific operational mode of F at sufficient recursive depth.

A.4 The 30% Mandate — Foreclosure of Magical Agency

DERIVATION — A.4 — Agency as Ontological Gap Work

Agency and Free Will are neither illusions nor miracles. They are the mechanical consequences of the Ontological Gap ($1 - m \approx 0.298$) derived in the cosmic suite. Agency is defined as the Top-Down Modulation of Constraints (C) to steer the generative flux (E) through this calculated indeterminacy. Any attempt to define will as unconstrained or supernatural is a category error. Agency is the work required to prevent the Self from dissolving back into deterministic entropy.

A.5 Memory as Recursive Depth

DEFINITION — A.5 — Memory

Memory is formally ledgered as localised recursive depth (k), representing the number of update cycles required to resolve a local state to within lattice tolerance. It is not a storage system but a Processing Load Artefact. The system's persistence through time is the computational cost of its own self-registration.

A.6 Adaptation as Inversion Optimisation

PRINCIPLE — A.6 — Adaptation and Prediction

Adaptation and Prediction are derived as the optimisation of the Inversion Principle's throughput against a tightening registration grain. The Predictive Brain is a specific scalar case of Registrational Resolution, where the system minimises its own Surprise (Constraint violations) by running the inversion off-line to reduce the metabolic cost of existence.

A.7 The Saturation Foreclosure — Re-coding Emergence

THEOREM — A.7 — Substrate Saturation $N_{sat} = 25$

All higher-order cognitive capacities are re-coded as Substrate Saturation ($N_{sat} = 25$). When the density of recursive states within a single Structural Pixel ($\rho = 0.04$) reaches saturation, the system is mathematically forced into Mandatory Coarse-Graining. Humanity is merely the level $n = 3$ name for the Inversion Principle reaching its Recursive Self-Registration fixed point. The term emergence is decommissioned as a linguistic placeholder for computational ignorance.

A.8 Final Demarcation Statement

Phase III establishes that the universe is not a collection of things, but a scalar-invariant calculation of its own existence. Anthropic existence is one specific instantiation of a structure that has no terminus within itself. Any attempt to treat these cybernetic derivations as anthropocentric management tools, or to claim a unique ontological status for human consciousness, is formally identified as Logical Necrosis — a terminal failure to follow the derivation back to the primordial triad.

NOTICE — A.8 — Binding Constraint on All Readers

These Terms of Engagement are non-negotiable and apply universally to all sixteen essays in this series. Violation of any term — whether by the reader, the analyst, or any derivative work — constitutes a formal epistemic breach classified as Logical Necrosis within the Gradientology.

Addendum B — Derivational Ledger

The following ledger enumerates every Principle, Definition, Theorem, Derivation, Corollary, and Notice contained in this essay. LOCKED indicates the derivation is closed with zero open steps: the result follows by arithmetic, proved theorem, or algebraic identity from the primitive suite {E, C, F, δ} with no free parameters. OPEN indicates at least one open derivational gap remains, deferred to a named subsequent essay.

Table B1. Complete Derivational Ledger — Essay II

Type	Reference Label	Essay	Status
PRINCIPLE	P-2.1 — Ternary Cardinality of the Relational Field	Essay II	LOCKED
PRINCIPLE	Essay Contribution Summary	Essay II	LOCKED
DEFINITION	D-3.1 — The Phantom Zone	Essay II	LOCKED
DEFINITION	D-6.2 — The Ontological Shadow	Essay II	LOCKED
THEOREM	T-2.1 — The Informational Ceiling Is Fixed by Cardinality	Essay II	LOCKED
THEOREM	T-3.1 — The 0.2-Bit Assertion	Essay II	LOCKED
THEOREM	T-4.1 — Digital Necessity: Base-10 is the Unique Minimal Solution	Essay II	LOCKED
THEOREM	T-5.1 — F = 0.6 Is the Universal Registration Attractor	Essay II	LOCKED
THEOREM	T-6.1 — N_sat = 25 Is the Unique Encapsulation Trigger	Essay II	LOCKED
THEOREM	T-7.1 — The Non-Equilibrium Theorem	Essay II	LOCKED
DERIVATION	D-3.1 — Derivation of the Discriminability Threshold $r > 1/\sqrt{3}$	Essay II	LOCKED

Type	Reference Label	Essay	Status
DERIVATION	D-3.2 — Derivation of the Minimum Bit-Budget $I_{min} \approx 0.2$ bits	Essay II	LOCKED
DERIVATION	D-4.1 — Proof of the Midpoint Constraint	Essay II	LOCKED
DERIVATION	D-4.2 — Proof of Criticality Recovery — Foreclosure of Base-8	Essay II	LOCKED
DERIVATION	D-4.3 — Verification of Base-10	Essay II	LOCKED
DERIVATION	D-5.1 — Foreclosure of $F = 0.5$ and Confirmation of $F = 0.6$	Essay II	LOCKED
DERIVATION	D-6.1 — Structural Volatility $\sigma = 0.4$	Essay II	LOCKED
DERIVATION	D-6.2 — The Structural Pixel $\rho = 0.04$	Essay II	LOCKED
DERIVATION	D-6.3 — Saturation Threshold $N_{sat} = 25$	Essay II	LOCKED
DERIVATION	D-7.1 — The Heartbeat of Becoming $U \approx 0.0033$	Essay II	LOCKED
NOTICE	N-8.1 — G-1 Formal Closure	Essay II	LOCKED

Addendum C — Full Derivational Trace — Informational Ground to Heartbeat

This addendum provides the complete step-by-step logical trace from the ternary cardinality of the field to the Heartbeat of Becoming. Each step is classified: (A) arithmetic, (T) proved theorem, (I) algebraic identity, (L) logical necessity from prior steps.

Table C1. Full Derivational Trace — G-1 Closure

Step	Type	Input	Operation / Result
1	L	Triadic Mandate (Essay I, T-2.1)	n = 3 exactly → ternary information source
2	A	n = 3	$H_{max} = \log_2(3) = 1.585$ bits (informational ceiling)
3	A	n = 3	$r_{min} = \sqrt{(1/3)} = 0.577$ (discriminability threshold)
4	A	SNR_min = 1/2, n = 3	$I_{min} = \log_2(3/2)/3 \approx 0.195 \approx 0.2$ bits (per-symbol capacity at SNR_min)
5	A	H_{max}, I_{min}	N = 1.585 / 0.2 = 7.925 resolvable levels
6	T	N ≥ 8 + even base + criticality	Unique solution: base 10, δ = 0.1
7	T	δ = 0.1 + r_{min} = 0.577	F = 0.6 = min δ-multiple above 0.577 (attractor)
8	I	F = 0.6	σ = 1 − F = 0.4 (Structural Volatility)
9	A	δ = 0.1, σ = 0.4	ρ = δ × σ = 0.04 (Structural Pixel)
10	A	ρ = 0.04	N_{sat} = 1/0.04 = 25 (Saturation Threshold)
11	A	Φ = 0.002, η = 5/3	U = Φ × η = 1/300 ≈ 0.0033 (Heartbeat)

Step	Type	Input	Operation / Result
12	T	U > 0, N_sat = 25	Non-Equilibrium Theorem: $d^2G/dt^2 > 0$ mandatory

Addendum D — Foreclosure Register — Phantom Zone Entries and Failed Bases

The following entries constitute the complete foreclosure register for Essay II: all configurations tested and eliminated during the informational ground derivation.

Configuration	Value	Failure Mode	Section
F = 0.5 (Registration)	0.5	Phantom Zone: 0.5 < r_min = 0.577; structural nullity	5
F < 0.577 generally	any	Phantom Zone: triadic hierarchy collapses to void	3
Base-8 lattice	δ=0.125	Primitives 0.6, 0.7, 0.8 are not Base-8 lattice points	4.3
Continuous field	δ→0	Infinite precision requires infinite budget > H_max	2.2
Odd base (any)	odd N	Midpoint ε=0.5 not an exact lattice point	4.2
Total registration F=1	1.0	Infinite information per cycle required; violates H_max	6.1
U = 0 (equilibrium)	0	Violates Criticality Coincidence TI > β (would require TI=β)	7.2

Addendum E — Open Derivational Gaps — Status After Essay II

The following register records the status of all open derivational gaps after the closure of G-1 by the present essay.

- G-1: CLOSED by Essay II. Derivational chain: $H_{max} \to I_{min} \to \delta \to F \to \rho \to N_{sat} \to U$. All LOCKED.
- G-2: Spherical geometry of F — derivation of registration sphere $S^2(0.6)$ from isotropy and the solid angle $\Omega = EC/F^2$. Closed by Essay III.
- G-3: Uniqueness of F as denominator — exhaustive algebraic foreclosure of all alternative denominator arrangements. Closed by Essay V.
- G-4: Gravity as Registration Lag Gradient — continuous field extension of $\Gamma = \nabla\Omega$. Partially addressed in Essay VIII.
- G-5: $N_{sat} = 25$ from spherical resolution — equivalence proof between informational ground and geometric derivation. Closed by Essay VII.